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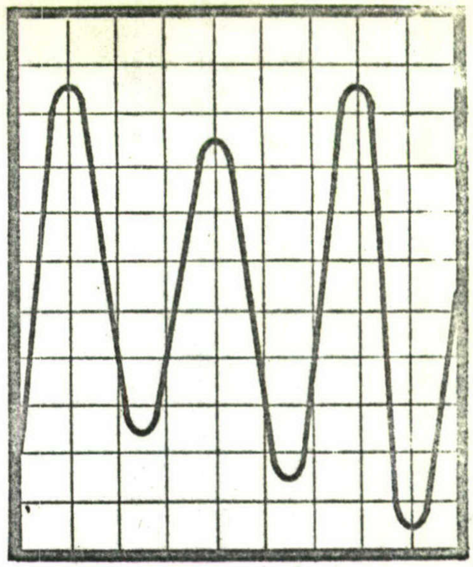

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# ESTIMATION OF THE PARAMETERS OF THE WEIBULL DISTRIBUTION USING THE METHOD OF LEAST SQUARE*

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The Weibull distribution is defined by the equation

$$F = F(t) = 1 - \mathrm{EXP} - (t - \gamma)^{\beta}/\eta \tag{1}$$

where $F(t)$ gives the cumulative percentage of failures up to time $t$; $t(t \geq \gamma \geq 0)$ is the part life; $\gamma$ is the location parameter (time before which no failures occur); $\eta(\eta > 0)$ is the characteristic life and $\beta(\beta > 0)$ is the shape parameter.

Dr. John H. K. Kao has described a now well-known graphical method for estimating the location parameter, as well as the shape and characteristic life parameters.

In this paper a method for estimating these parameters by the use of least squares is given.

It is noted that with double logarithmic transformation, Equation (1), upon making the following substitutions, assumes the form of a straight line equation:

$$Y = aX + b \tag{2}$$

where

$$Y = \mathrm{LnLn}\,(1 - F)^{-1} \tag{3}$$

$$X = \mathrm{Ln}\,(t - \gamma) \tag{4}$$

$$a = \beta \tag{5}$$

$$b = -\beta\,\mathrm{Ln}\,\eta \tag{6}$$

This transformation leads to the use of least squares for estimating each of the parameters. Solution is achieved by taking the partial derivatives of the following equation:

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$$G = \sum_{i=1}^{N} [Y_i - (aX_i + b)]^2 \tag{7}$$

with respect to each parameter and setting the equations equal to zero. Further, define

$$\text{LNLN} \frac{1}{1 - F} = D$$

$$\text{LN}(t - \gamma) = E$$

$$(t - \gamma)^{-1} = F$$

Now, by solving simultaneously, the equations for each parameter become

$$\Sigma D = \frac{\{\Sigma D/(t - \gamma)\}\{N\Sigma E^2 - (\Sigma E)^2\} + \{\Sigma DE\}\{(\Sigma F)(\Sigma E) - N\Sigma EF\}}{\{(\Sigma F)(\Sigma E^2)\} - \{\Sigma FE\}\{\Sigma E\}} \tag{8}$$

$$\beta = \frac{N\Sigma DE - \Sigma E\Sigma D}{N\Sigma E^2 - (\Sigma E)^2} \tag{9}$$

$$\text{LN}\eta = \frac{(\Sigma DE)\Sigma E - (\Sigma E^2)\Sigma D}{N\Sigma DE - (\Sigma D)(\Sigma E)} \tag{10}$$

where the summations are taken over the set of sample data.

(The shape and characteristic life parameters have previously been estimated using the method of least squares, provided the location parameter is known or is set equal to zero.)

Note that Equation (8) is an implicit function with $\gamma$ as the only unknown. After solving for $\gamma$, $\beta$ and $\eta$ can be solved for directly using equations (9) and (10), respectively. For a high degree of accuracy, a digital computer should be used.

## REFERENCES

1. Weibull, Waloddi, "A Statistical Distribution Function of Wide Applicability," Journal of Applied Mechanics, September 1951, Pages 293 - 297.

2. Kao, John H. K., "A Summary of Some New Techniques on Failure Analysis," Proceedings, Sixth National Symposium on Reliability and Quality Control in Electronics, Washington, D. C., January 11 - 13, Pages 190 - 201.